There is an identity involving $\sinh x$ and $\cosh x$ that resembles a Pythagorean identity from trigonometry. **SCORE: ______ / 4 PTS**

[a] Write that identity involving $\sinh x$ and $\cosh x$. **NOTE: You do NOT need to prove the identity.**

$$\cosh^2 x - \sinh^2 x = 1 \quad (1)$$

[b] Divide both sides of that identity by $\cosh^2 x$ and simplify.

$$1 - \tanh^2 x = \operatorname{sech}^2 x \quad (1)$$

[c] If $\sinh x = -7$, find $\coth x$.

$$\cosh^2 x - (-7)^2 = 1$$
$$\cosh^2 x = 50 \quad \left(\tfrac{1}{2}\right)$$

EITHER IS OK

$$\cosh x = \pm 5\sqrt{2} \quad \left(\tfrac{1}{2}\right)$$

SINCE $\cosh x > 0$, $\left(\tfrac{1}{2}\right)$

THEREFORE $\cosh x = 5\sqrt{2}$ $\left(\tfrac{1}{2}\right)$

$$\coth x = \frac{\cosh x}{\sinh x} = -\frac{5\sqrt{2}}{7} \quad \left(\tfrac{1}{2}\right)$$

Write and **prove** a formula for $\sinh(x-y)$ in terms of $\sinh x$, $\sinh y$, $\cosh x$ and $\cosh y$. **SCORE: ______ / 6 PTS**

$$\boxed{\sinh x \cosh y - \cosh x \sinh y} \quad 2$$

$$= \boxed{\frac{e^x - e^{-x}}{2} \cdot \frac{e^y + e^{-y}}{2} - \frac{e^x + e^{-x}}{2} \cdot \frac{e^y - e^{-y}}{2}} \quad 1$$

$$= \boxed{\frac{\cancel{e^{x+y}} + e^{x-y} - e^{y-x} - \cancel{e^{-x-y}} - (\cancel{e^{x+y}} - e^{x-y} + e^{y-x} - \cancel{e^{-x-y}})}{4}} \quad 2$$

$$= \frac{2e^{x-y} - 2e^{y-x}}{4}$$

$$= \boxed{\frac{e^{x-y} - e^{-(x-y)}}{2}} \quad 1$$

$$= \sinh(x-y)$$

Prove that $g(x)=\ln(x+\sqrt{x^2-1})$ is the inverse of $f(x)=\cosh x$ by simplifying $g(f(x))$. **SCORE: ______ / 5 PTS**

$\ln(\cosh x+\sqrt{\cosh^2 x-1})$ 1

$=\ln(\cosh x+\sqrt{\sinh^2 x})$

$=\ln(\cosh x+\sinh x)$ 1

$=\ln\left(\frac{e^x+e^{-x}}{2}+\frac{e^x-e^{-x}}{2}\right)$ 1

$=\ln\frac{2e^x}{2}$

$=\ln e^x$ 1

$=x$ 1

Solve $\sinh x = 1$ by using the exponential definition of $\sinh x$ and an algebraic substitution $z = e^x$. **SCORE: ______ / 6 PTS**

$$\frac{e^x - e^{-x}}{2} = 1$$

$$\frac{z - \frac{1}{z}}{2} = 1 \quad 1$$

$$z - \frac{1}{z} = 2$$

$$z^2 - 1 = 2z$$

$$z^2 - 2z - 1 = 0 \quad 1$$

$$z = \frac{2 \pm \sqrt{4+4}}{2}$$

$$= \frac{2 \pm \sqrt{8}}{2} \quad \tfrac{1}{2}$$

EITHER IS OK

$$= \frac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2} \quad 1$$

$$z = e^x > 0 \quad 1$$

SO $z = 1 + \sqrt{2}$

$$e^x = 1 + \sqrt{2}$$

$$x = \ln(1 + \sqrt{2}) \quad 1\tfrac{1}{2}$$

Rewrite $\operatorname{csch}(3\ln 2)$ in terms of exponential functions and simplify. **SCORE: ______ / 3 PTS**

$$\boxed{\frac{2}{e^{3\ln 2}-e^{-3\ln 2}}} = \frac{2}{e^{\ln 8}-e^{\ln \frac{1}{8}}} = \boxed{\frac{2}{8-\frac{1}{8}}} = \boxed{\frac{16}{63}}$$

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